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NORMAL LIMITS IN STAR-INVARIANT SUBSPACES IN MULTIPLY CONNECTED DOMAINS

JAWAD SADEK

ABSTRACT. In this paper we give a necessary and sufficient condition for the normal limit to exist at a point on the smooth boundary of a multiply connected domain in the plane, for a function in the star-invariant subspace.

Introduction. Let U be the open unit disc in the complex plane, and denote by H^p, $0 < p \leq \infty$, the usual classes of analytic functions on U [**9, 13, 12**]. Let φ be an inner function and write $\varphi = cBS_\sigma$ where $|c| = 1$, B is a Blaschke product with zero sequence $\{z_k\}$, and S_σ is a singular inner function with positive measure σ which is singular with respect to Lebesgue measure [**9, 13, 12**].

In [**2**, Lemma 3] Ahern and Clark gave the following generalization of a famous theorem of Frostman [**10**] concerning the existence of the radial limit of a Blaschke product at a given point in T, the unit circle.

Theorem A. *Let ζ_0 be on the unit circle T, and suppose $\varphi = BS_\sigma$ and $\sigma(\{\zeta_0\}) = 0$. Then the following conditions are equivalent*:

(i) *Every divisor of φ has a radial limit of modulus 1 at ζ_0.*

(ii) *Every divisor of φ has a radial limit at ζ_0.*

(iii) $\sum_{k=1}^{\infty}(1-|z_k|)/|\zeta_0 - z_k| + \int_T d\sigma(u)/|u-\zeta_0| < \infty$.

We say that f is a divisor of φ if $\varphi = fg$ where both f and g lie in the unit ball of H^∞.

In [**6**, Theorem 3.1], Cohn noticed that condition (iii) implies a stronger result than (ii). Let φ be an inner function, and let

$$K_2 \equiv H^2 \ominus \varphi H^2$$

be the star-invariant subspace generated by φ. Let $BMOA$ denote the space of analytic functions of bounded mean oscillation and define $K_* \equiv K_2 \cap BMOA$. Then we have the following result.

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