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A FURTHER APPLICATION OF THE deLEEUW-GLICKSTEIN THEOREM

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ABSTRACT. The theorem of Ionescu Tulcea and Marinescu, a generalization of the ergodic theorem of Doeblin and Fortet, is derived as a result of the deLeeuw-Glicksberg decomposition. This complements recent work of Sine which derives asymptotic periodicity results for constricted Markov operators from the deLeeuw-Glickstein decomposition. A weak form of the Ionescu Tulcea and Marinescu theorem is obtained from the deLeeuw-Glickstein decomposition.

1. Introduction. A discrete time parameter Markov chain with stationary transition probabilities and state space E is specified by a transition function $P(x, A)$ on $E \times \mathcal{E}$, where $\mathcal{E}$ is a σ-algebra of subsets of E; or equivalently, the Markov operator P associated with $P(x, A)$ on some suitable function space X. A large portion of the theory of Markov chains deals with the asymptotic behavior of the iterates P^n of P as $n \to \infty$. Over the years, the accumulated evidence has been that, under certain not unrealistic general conditions, the iterates $P^n f$ approach a finite dimensional space which is either independent of f or, if not, dependent in an obvious way; we will refer to this as *asymptotic periodicity.* Asymptotic periodicity has been demonstrated in one form or another for distance-diminishing operators [**3**], quasicompact operators [**2** and others since], uniform mean stable operators [**16, 8, 17**], and lately constricted operators [**12, 9, 10**].

A technique which is emerging as a useful tool is the deLeeuw-Glicksberg decomposition [**1**], which will be described now for future reference. Let X be a real or complex Banach space, let X' be the dual space, the continuous linear functionals on X, and let $[X]$ denote the set of all bounded linear operators $T : X \to X$. In addition to the strong, i.e., norm, topology of X, we shall also be interested in the weak topology in X. When X is considered with its strong topology, we shall consider $[X]$ with the strong operator topology; when X is considered with its weak topology, we shall take $[X]$ to be equipped with the weak

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