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THE DUAL OF BERGMAN METRIC VMO

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ABSTRACT. The space BMO_p denotes the variant of BMO based on balls of constant size in the Bergman metric on a strongly pseudoconvex domain, with the mean oscillation measured in an L^p sense. It is closely connected to the study of Hankel operators on the Bergman space which are bounded in the L^p norm. This paper presents a correct proof that the dual of the Bergman metric VMO_p is the space H_q^1 consisting of l^1-sums of Bergman metric q-atoms, $1/p + 1/q = 1$; and that the second dual is BMO_p. In the course of the proof, it is shown that the linear functional corresponding to a sum of atoms is independent of this decomposition into atoms, and an intrinsic formula for the duality pairing (independent of the decomposition) is derived.

1. Introduction. I am writing this paper mainly to correct an oversight of mine in the paper, "BMO on strongly pseudoconvex domains: Hankel operators, duality and $\bar{\partial}$-estimates," [**3**], by Huiping Li and me. In that paper, in Theorem 4.5, is the claim that the dual of the space VMO_p, $p > 1$, is the space called H_q^1, where $1/p + 1/q = 1$, and that the dual of H_q^1 is BMO_p (definitions in Section 2). These statements are indeed true, but the proof presented there (due entirely to me) is at best incomplete.

The proof presented in [**3**] makes the claim that the dual of VMO_p is entirely representable as l^1 sums of q-atoms, citing "standard functional analysis arguments" without actually exhibiting them. I did in fact have in mind a standard technique, but unfortunately it was one that did not apply to that situation! I had incorrectly reversed the roles of a Banach space and its dual. There were some less serious errors of omission as well: all essentially the omission of a verification that some mapping was well-defined. The erroneous proof and these omissions will be corrected here in Section 3.

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