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A CHARACTERIZATION OF FGC RINGS

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ABSTRACT. In this note we give a new characterization of commutative rings for which every finitely generated module is a direct sum of cyclics (FGC rings) using only the structure of the injective envelopes of simple modules. Some Baer-Kaplansky categories for FGC rings are studied.

1. Introduction. The structure of commutative rings for which every finitely generated module is a direct sum of cyclics was determined more than twenty-five years ago as the culmination of the work of a number of mathematicians over many years. A self contained exposition of the proof is given in [**14**]. The characterization is internal but it relies on the structure of almost maximal valuation rings for which Gill ([**3**]) obtained a characterization in terms of their indecomposable injectives. An analogous characterization of noncommutative serial rings whose finitely generated modules are direct sums of uniserials (the noncommutative analogue of almost maximal valuation rings) was obtained in [**4**]. It may therefore be useful for the study of noncommutative analogues of FGC rings to have a characterization of FGC rings in terms of their injectives. This note does that. The proof is obtained by reducing to the structure theorem obtained in [**14**]. It would therefore be valuable to have a direct proof that the rings satisfying the characterization are FGC rings.

There has been considerable interest in categories of modules which are determined by their endomorphism rings since the pioneering work of Baer [**1**] and Kaplansky [**6**]. (We refer the reader to [**8**] for a survey of this and related areas and an extensive bibliography of nearly 300 items.) However, it was only recently that Baer-Kaplansky categories for virtually arbitrary rings were shown to exist [**5**]. We obtain some Baer-Kaplansky categories of modules for arbitrary FGC rings and thus show that in this sense, also, FGC rings are generalizations of the ring of integers **Z**.

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