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A CLASS OF ABELIAN GROUPS DEFINED BY CONTINUOUS CROSS SECTIONS IN THE BOHR TOPOLOGY

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ABSTRACT. Comfort, Hernández and Trigos-Arrieta [**2**] introduced the class **ACCS**(#) of abelian groups H such that the natural map $\varphi : G \to G/H$, where G is the divisible hull of H, has a cross section $\Gamma : G/H \to G$ that is *continuous* in the Bohr topology of G and G/H. They showed that **ACCS**(#) is closed under finite products and contains all finitely generated groups (and, of course, all divisible groups). They also gave an example of a group that does not belong to **ACCS**(#). We give further examples of groups from **ACCS**(#) (e.g., the groups of p-adic integers) and we find some new restraints for the groups from **ACCS**(#). This entails that large powers of nondivisible abelian groups never belong to **ACCS**(#) and gives an upper bound for the size of the reduced groups in **ACCS**(#) (roughly speaking, most of the abelian groups do not belong to **ACCS**(#)).

1. Introduction. The *Bohr topology* of an abelian group G is the initial topology on G with respect to the family of all homomorphisms of G into the circle group. Following van Douwen [**6**], we write $G^{\#}$ for an abelian group G equipped with its Bohr topology.

E.K. van Douwen [**6**] (cf. [**1**, p. 515]) raised the following question: *Are $G^{\#}$ and $H^{\#}$ homeomorphic as topological spaces whenever G and H are abelian groups of the same size?* A negative answer to this question was given independently and around the same time in [**11**], [**5**]. On the other hand, it was proved recently by Comfort, Hernández and Trigos-Arrieta [**2**] that $\mathbf{Q}^{\#}$ and $\mathbf{Z}^{\#} \times (\mathbf{Q}/\mathbf{Z})^{\#} = (\mathbf{Z} \times (\mathbf{Q}/\mathbf{Z}))^{\#}$ are homeomorphic. The proof of this quite surprising fact is related to another question of van Douwen.

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