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HILBERT-SIEGEL MODULI SPACES IN POSITIVE CHARACTERISTIC

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Hilbert-Siegel varieties are moduli spaces for abelian varieties equipped with an action by an order O_K in a fixed, totally real field K. As such, they include both the Siegel moduli spaces (use $K = \mathbf{Q}$ and the action is the standard one) and Hilbert-Blumenthal varieties (where the dimension of K is the same as that of the abelian varieties in question). In this paper we study certain phenomena associated to Hilbert-Siegel varieties in positive characteristic. Specifically, we show that ordinary points are dense in moduli spaces of mildly inseparably polarized abelian varieties with action by a given totally real field. Moreover, we introduce a combinatorial invariant of the first cohomology of an abelian variety which allows us to compute and explain the singularities of such a moduli space.

The problem considered here arises in two distinct but closely related lines of inquiry. On one hand, recall that if X is an abelian variety over a field k of characteristic p, then its p-torsion is described by $X[p](\bar{k}) \cong (\mathbf{Z}/p\mathbf{Z})^\rho$ for some ρ. This integer ρ, the p-rank, is between zero and $\dim X$. When ρ is maximal, the abelian variety is said to be ordinary. Deuring shows that the generic elliptic curves is ordinary [**4**]. Mumford announces [**13**], and Norman and Oort prove [**15**], the obvious generalization of this statement to higher dimension: ordinary points are dense in the moduli space of polarized abelian varieties. Wedhorn has recently obtained similar results [**19**] for families of principally polarized abelian varieties with given ring of endomorphisms.

On the other hand, moduli spaces of PEL type—those parametrizing abelian varieties with certain polarization, endomorphism and level-structure data—are important spaces in their own right. Roughly speaking, when the characteristic of the ground field is relatively prime to the moduli problem, the resulting space is smooth. When the characteristic resonates with the moduli functor, things get interesting and

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