ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 33, Number 3, Fall 2003

OSCILLATION AND NONOSCILLATION THEOREMS FOR FOURTH ORDER DIFFERENCE EQUATIONS

EWA SCHMEIDEL

ABSTRACT. This paper is concerned with a class of fourth order nonlinear difference equations. Two types of nonoscillatory solutions will be considered. Relations between these types of solutions and their oscillatory behavior are the main purpose of this paper.

1. Introduction. In several papers the oscillatory and asymptotic behavior of solutions of third order difference equations have been discussed. For example, note the papers [**12**]–[**15**] and [**17**]. When compared to differential equations the study of difference equations has received little attention for orders greater than three. Fourth order linear difference equations are considered in [**4**], [**6**], [**16**], [**18**] and [**19**]. Fourth order nonlinear difference equations are studied in [**10**], [**11**] and [**20**].

In this paper we will study fourth order difference equations

$$\text{(E)} \qquad \Delta^4 y_n = f(n, y_n), \quad n \in N = \{0, 1, 2, \dots\}$$

where Δ is the forward difference operator $\Delta y_n = y_{n+1} - y_n$ and $\Delta^k y_n = \Delta(\Delta^{k-1} y_n)$ for $k = 2, 3, \dots$. The sequence $y = \{y_n\}$ is the trivial sequence if there exists $n_0 \in N$ such that $y_n = 0$ for all $n \geq n_0$. By a solution of (E), we mean any nontrivial sequence $\{y_n\}$ satisfying equation (E), for all $n \in N$. In general, we will assume that the usual existence and uniqueness theorem for solutions of equation (E) holds. A solution is oscillatory if, for every $m \in N$, there exists $n \geq m$ such that $y_n y_{n+1} \leq 0$. Therefore a nonoscillatory solution is eventually positive or eventually negative. We assume that void sum is equal to zero. In the paper we assume that this function $f : N \times R \to R$ satisfies condition

$$(*) \qquad x f(n, x) < 0 \quad \text{for } n \in N,\ x \in R \setminus \{0\}.$$

1991 AMS *Mathematics Subject Classification.* 39A10.
Key words and phrases. Difference equation, fourth order, oscillatory and nonoscillatory solutions.
Received by the editors on September 1, 2000, and in revised form on July 11, 2001.