

LINEAR MAPS BETWEEN BANACH ALGEBRAS COMPRESSING CERTAIN SPECTRAL FUNCTIONS

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ABSTRACT. In this paper, we discuss the linear maps between semi-simple Banach algebras which compress any one of the spectrum, the left spectrum, the right spectrum, the intersection of left spectrum and right spectrum, the boundary of spectrum and the full spectrum. We prove that such linear maps are idempotent preserving. As applications, we characterize such maps in terms of Jordan homomorphisms on C^* -algebras of real rank zero. In particular, we give several characterizations of isomorphisms between standard operator algebras by using such spectral function compressing linear maps and surjectivity spectrum compressing or approximate point spectrum compressing linear maps.

1. Introduction. Over the past decade, there has been a considerable interest in the study of linear maps on operator algebras that preserve certain properties of operators. In particular, a problem of how to characterize linear maps that preserve the spectrum of each operator has attracted the attention of many mathematicians. In [16], Jafarian and Sourour proved that a surjective linear map preserving spectrum from $\mathcal{B}(X)$ onto $\mathcal{B}(Y)$ is either an isomorphism or an anti-isomorphism, where X and Y are complex Banach spaces, and $\mathcal{B}(X)$ is the Banach algebra of all bounded linear operators acting on X . Aupetit and Mouton [3] extended the result of Jafarian and Sourour to primitive Banach algebras with minimal ideals. In [21], Sourour characterized the linear bijective maps preserving invertibility from $\mathcal{B}(X)$ onto $\mathcal{B}(Y)$ and obtained a similar result when linear map is unital. Bresar and Semrl [6] proved that a linear surjective map preserving spectral radius on $\mathcal{B}(X)$ is either an automorphism or an anti-automorphism multiplied by a scalar with modulus 1. It is shown in [20] that every point spectrum

1991 AMS *Mathematics Subject Classification.* Primary 47D25, 47B49, 47A10.
Key words and phrases. Jordan homomorphism, isomorphism, linear preservers, Banach algebras, C^* -algebras.

This work is supported by NNSFC, PNSFS, PYSFS and China Postdoctoral Science Foundation.

Received by the editors on November 13, 2001, and in revised form on February 23, 2002.