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ON THE STRONG LAW FOR ASYMPTOTICALLY ALMOST NEGATIVELY ASSOCIATED RANDOM VARIABLES

TAE-SUNG KIM, MI-HWA KO AND IL-HYUN LEE

ABSTRACT. In this paper the Hajeck-Renyi type inequality for asymptotically almost negatively associated (AANA) random variables is derived and the strong law of large numbers is obtained by applying this inequality. The strong laws of large numbers for weighted sums of AANA random variables are also considered.

1. Introduction. Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a probability space and $\{X_1, \dots, X_n\}$ a sequence of random variables defined on $(\Omega, \mathcal{F}, \mathcal{P})$. A finite family $\{X_i,\, 1 \le i \le n\}$ is said to be negatively associated (NA) if for any disjoint subsets $A, B \subset \{1, \dots, n\}$ and any real coordinatewise nondecreasing functions $f : R^A \to R$ and $g : R^B \to R$,

$$\operatorname{Cov}\,(f(X_i;\, i \in A), g(X_j;\, j \in B)) \le 0.$$

Infinite family of random variables is negatively associated (NA) if every finite subfamily is negatively associated (NA). This concept was introduced by Joag-Dev and Proschan [**8**]. A sequence $\{X_n,\, n \ge 1\}$ of random variables is called asymptotically almost negatively associated (AANA) if there is a nonnegative sequence $q(m) \to 0$ such that

$$\begin{aligned}(1)\quad &\operatorname{Cov}\,(f(X_m), g(X_{m+1}, \dots, X_{m+k})) \\ &\qquad \le q(m)(\operatorname{Var}\,(f(X_m))\operatorname{Var}\,(g(X_{m+1}, \dots, X_{m+k})))^{1/2}\end{aligned}$$

for all $m, k \ge 1$ and for all coordinatewise increasing continuous functions f and g whenever the righthand side of (1) is finite. This definition was introduced by Chandra and Ghosal [**2**, **3**].

The family of AANA sequences contains negatively associated (in particular, independent) sequences (with $q(m) = 0$ for all $m \ge 1$)

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