

WEAK BOURBAKI UNMIXED RINGS: A STEP TOWARDS NON-NOETHERIAN COHEN-MACAULAYNESS

TRACY DAWN HAMILTON

ABSTRACT. Weak Bourbaki unmixed rings are defined in this paper. The definition of a weak Bourbaki unmixed ring is a candidate for an “appropriate” definition of Cohen-Macaulayness. We will see that this definition satisfies many of the conditions we want an “appropriate” definition to satisfy. It is not yet known whether this definition (or any other) satisfies all of the conditions. However, no example has been found of a weak Bourbaki unmixed ring which violates one of the conditions.

1. Introduction. Cohen-Macaulay rings have been studied since 1916 when Macaulay’s *The algebraic theory of modular systems* [10] was published. At the time, Macaulay was dealing with the question of describing solutions to sets of polynomial equations. Since that time the theory of Cohen-Macaulay rings (and modules) has grown to be one of the major areas of study in commutative algebra and in algebraic geometry, see [1].

It was not until 1992 that anyone brought up the idea of extending the concept of Cohen-Macaulayness to non-Noetherian rings. In 1992, Glaz published a paper in which she studied fixed rings [3]. At the end of that paper there was a conjecture that under certain conditions these fixed rings would be Cohen-Macaulay. However, since the rings Glaz was studying were not Noetherian, this conjecture cannot be considered until there is an appropriate definition of non-Noetherian Cohen-Macaulayness. In fact, Glaz goes on to say that the “first step toward solving this conjecture is finding the right definition of non-Noetherian Cohen-Macaulayness” [3].

In 1994 [4], Glaz refined this question by asking whether one can define a non-Noetherian Cohen-Macaulay ring so that:

Received by the editors on October 27, 2000, and in revised form on October 26, 2001.