

SHARPENING HÖLDER'S AND POPOVICIU'S INEQUALITIES VIA FUNCTIONALS

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ABSTRACT. We prove some inequalities involving positive isotonic linear functionals which generalize Hölder's inequality and its reverse version. We also sharpen Jensen's inequalities for positive isotonic linear functionals.

1. Introduction. In the articles [1] and [2] sharpenings of the integral versions of Hölder's and Jensen's inequalities were obtained. Here we improve these results using a positive isotonic functional leading to some new generalizations of Hölder's and Popoviciu's inequalities. The new results sharpen Hölder's and Popoviciu's inequalities and their reversed versions both in discrete and integral forms.

Let E be a nonempty set and L be a linear class of real-valued functions $f : E \rightarrow R$ having the properties:

L1. $f, g \in L \Rightarrow (af + bg) \in L$ for all $a, b \in R$;

L2. $1 \in L$, i.e., if $f(t) = 1$ for all $t \in E$, then $f \in L$.

Let A be a positive isotonic linear functional on L . That is, we assume that

A1. $A(af + bg) = aA(f) + bA(g)$ for $f, g \in L$, $a, b \in R$ (linearity);

A2. $f \in L$, $f(t) \geq 0$ on $E \Rightarrow A(f) \geq 0$ (positive isotonic).

Functional versions of well-known inequalities and related results could be found in [10]. Here, we mention results related to Jensen's inequality.

Theorem A [10, p. 112] (Jensen's inequality). *Let L satisfy conditions L1, L2 and A satisfy conditions A1 and A2. Suppose that $k \in L$*

This work was done while the third-named author was visiting the University of Haifa in Israel. She gratefully acknowledges the hospitality of the University of Haifa.

Key words and phrases. Convex function, concave function, Hölder's inequality, Jensen's inequality, Popoviciu's inequality, positive isotonic linear functional.

Received by the editors on February 28, 2000, and in revised form on April 10, 2002.