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CONVOLUTION AND FOURIER TRANSFORM OVER THE SPACES $\mathcal{K}'_{p,k}$, $p > 1$

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ABSTRACT. We introduce the space $\mathcal{K}_{p,k}, p > 1$ that is the vector space of all C^∞-functions f such that $e^{k|x|^p}\partial^\alpha f$ vanishes at infinity for all $\alpha \in N^n$ and its dual $\mathcal{K}'_{p,k}$. For $f, g \in \mathcal{K}'_{p,2^p k}$, $k \in Z$, $k < 0$, we study the linear functional $f \otimes g$ on $\mathcal{K}_{p,k}$ defined by

$$\langle f \otimes g, \phi \rangle = \langle f(x), \langle g(y), \phi(x+y) \rangle \rangle, \quad \phi \in \mathcal{K}_{p,k}.$$

Also, we show a representation theorem and an inversion formula for the usual distributional Fourier transform over the spaces $\mathcal{K}'_{p,k}$, $k \in Z$, $k < 0$.

1. Introduction. For spaces of functions and distributions we use the notations and terminology of Horvath [**3**]. In particular, $\mathcal{S}_k$ is the space of all infinitely differentiable functions f on R^n such that $(1 + |x|^2)^k \partial^\alpha f(x)$ vanishes at infinity for all $\alpha \in N^n$.

We denote $\mathcal{K}_p$, $p \geq 1$, the space of all functions $\phi \in C^\infty(R^n)$ such that

$$\nu_k(\phi) = \sup_{\substack{x \in R^n \\ |\alpha| \leq k}} e^{k|x|^p} |D^\alpha \phi(x)| < \infty, \quad k = 1, 2, \dots,$$

where $D^\alpha = (i^{-1}\partial/\partial x_1)^{\alpha_1} \cdots (i^{-1}\partial/\partial x_n)^{\alpha_n}$ and $|\alpha| = \alpha_1 + \cdots + \alpha_n$. The space $\mathcal{K}_p$ with semi-norm ν_k, $k = 1, 2, \dots$ is a Frechet space and the space of C^∞-functions with compact support $\mathcal{D}$ is a dense subset of $\mathcal{K}_p$. By $\mathcal{K}'_p$ we mean the space of continuous linear functionals on $\mathcal{K}_p$. For further details, we refer to [**4**].

We introduce the spaces $\mathcal{K}_{p,k}(R^n)$, $p > 1$, that are defined as the vector spaces of all functions f defined on R^n which possess continuous

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