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A-COMPACTNESS AND MINIMAL SUBALGEBRAS OF $C(X)$

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ABSTRACT. Let $\sum(X)$ be the set of all subalgebras of $C(X)$ containing $C^*(X)$, where X is a Tychonoff space. Given $A(X) \equiv A \in \sum(X)$ there is associated a subset $\upsilon_A X$ of βX which is an A-analogue of the Hewitt real compactification υX of X. X is called A-compact if and only if $\upsilon_A X = X$. Redlin and Watson asked whether, for any real compact space X, there exists in some sense a minimal $A_m \in \sum(X)$ for which X becomes A_m-compact. Acharyya, Chattopadhyay and Ghosh answered this question in affirmative by defining a suitable preorder on $\sum(X)$, and they made the following conjecture that there does not exist any minimal subalgebra $A(\mathbf{N})$ of $C(\mathbf{N})$ containing $C^*(\mathbf{N})$, in the usual set inclusion sense for which $\mathbf{N}$ is A-compact. In this paper we have shown that, given any real compact space X there does not exist any minimal member $A_m \in \sum(X)$, in the usual set inclusion sense for which X becomes A_m-compact and thereby proving the conjecture as a special case of it. From this result it has been further shown that for any $A(X) \neq C^*(X)$ in $\sum(X)$ there does not exist any minimal member $B(X) \in \sum(X)$ in the usual set inclusion sense for which $\upsilon_A X = \upsilon_B X$.

1. Introduction. It is a remarkable fact in the theory of rings of continuous functions that the Stone-Čech compactification βX of a Tychonoff space X could be realized as the set of all maximal ideals of an arbitrary subalgebra $A(X)$ of $C(X)$ containing $C^*(X)$, equipped with hull kernel topology. Such subalgebras of $C(X)$ were initiated and investigated in detail by Plank [**7**]. Let $\sum(X)$ be the family of all such subalgebras of $C(X)$. Following Redlin and Watson [**8**], for any $A(X) \equiv A \in \sum(X)$, a maximal ideal M of A is called *real* provided that the quotient field A/M is isomorphic to the real field $\mathbf{R}$, otherwise

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