

OSCILLATION RESULTS FOR LINEAR MATRIX HAMILTONIAN SYSTEMS

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ABSTRACT. In this paper we present new oscillation criteria in terms of the coefficient functions for the matrix linear Hamiltonian systems $X' = A(t)X + B(t)Y$, $Y' = C(t)X - A^*(t)Y$, which are not contained in our recent paper [15], and improve the main results in [15] to some extent.

1. Introduction. Consider the linear Hamiltonian system

$$(1.1) \quad \begin{cases} X' = A(t)X + B(t)Y \\ Y' = C(t)X - A^*(t)Y, \end{cases} \quad t \geq t_0$$

where $X(t)$, $Y(t)$, $A(t)$, $B(t)$, $C(t)$ are $n \times n$ real continuous matrix functions such that $B(t)$ and $C(t)$ are symmetric and $B(t)$ is positive definite, i.e., $B(t) > 0$ for $t \geq t_0$. By M^* we mean the transpose of the matrix M .

For any two solutions $X_1(t)$, $Y_1(t)$ and $X_2(t)$, $Y_2(t)$ of (1.1) the *Wronskian* $X_1^*(t)Y_2(t) - Y_1^*(t)X_2(t)$ is a constant matrix. In particular, for any solution $X(t)$, $Y(t)$ of (1.1), $X^*(t)Y(t) - Y^*(t)X(t)$ is a constant matrix. We now recall for the sake of convenience of reference the following definitions from the earlier literature.

Definition 1.1. A solution $X(t)$, $Y(t)$ of (1.1) is said to be nontrivial if $\det X(t) \neq 0$ for at least one $t \in [t_0, \infty)$.

Definition 1.2. A nontrivial solution $X(t)$, $Y(t)$ of (1.1) is said to be prepared if, for every $t \in [t_0, \infty)$,

$$(1.2) \quad X^*(t)Y(t) - Y^*(t)X(t) = 0.$$

2000 AMS *Mathematics Subject Classification*. Primary 34A30, 34C10.
Key words and phrases. Oscillation, linear, matrix Hamiltonian system.
Received by the editors on June 13, 2003, and in revised form on October 25, 2003.