ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 36, Number 1, 2006

APPROXIMATION OF INVARIANT MEASURES FOR RANDOM ITERATIONS

ANDERS ÖBERG

ABSTRACT. In this paper iterated function systems are investigated from the point of view of their invariant measures. Different ideas of how to approximate invariant measures are investigated, but we also discuss necessary and sufficient conditions for uniqueness of an invariant measure. We also consider sufficient conditions for the measure separated open set condition under weak assumptions.

1. Introduction. In this paper we generalize some results obtained in Strichartz et al. [**33**] from the interval to compact subsets of Euclidean spaces. In [**33**], they managed to approximate invariant measures associated to iterated function systems with finitely many maps S_j, that is measures which satisfy an identity of the form

$$\mu = \sum_{j=1}^{m} p_j \mu \circ S_j^{-1}, \quad m \geq 2. \tag{1.1}$$

In the part of [**33**] we are presently interested in, one has one-to-one maps $S_j : [0,1] \to [0,1]$ with strictly positive continuous probability weight functions p_j. In [**33**] a *non-overlapping condition* was also imposed, which was the key to their successful *approximation algorithm* and was also sufficient to establish *uniqueness* of an invariant measure of the form (1.1). If the state space is X, this non-overlapping condition means that the interiors of the images $S_j X$ are disjoint and if $X = K$, a compact subset of a Euclidean space, the so-called *open set condition*, see Hutchinson [**15**], is satisfied with the interior of K as the open set.

A simple condition for uniqueness of an invariant measure is strict contraction of the maps, see for instance [**15**, Theorem 1, p. 733]. There

2000 AMS *Mathematics Subject Classification.* Primary 60B10, Secondary 28A80, 37A30, 60J05.
Key words and phrases. Iterated function system, uniqueness of invariant measure, open set condition.
Received by the editors on July 12, 2003.