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ON THE LUPAŞ q-ANALOGUE OF THE BERNSTEIN OPERATOR

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ABSTRACT. Let $R_n(f, q; x) : C[0, 1] \to C[0, 1]$ be q-analogues of the Bernstein operators defined by Lupaş in 1987. If $q = 1$, then $R_n(f, 1; x)$ are classical Bernstein polynomials. For $q \neq 1$, the operators $R_n(f, q; x)$ are rational functions rather than polynomials. The paper deals with convergence properties of the sequence $\{R_n(f, q; x)\}$. It is proved that $\{R_n(f, q_n; x)\}$ converges uniformly to $f(x)$ for any $f(x) \in C[0, 1]$ if and only if $q_n \to 1$. In the case $q > 0$, $q \neq 1$ being fixed the sequence $\{R_n(f, q; x)\}$ converges uniformly to $f(x) \in C[0, 1]$ if and only if $f(x)$ is linear.

1. Introduction. In 1912 Bernstein ([**2**]) found his famous proof of the Weierstrass approximation theorem. Using probability theory he defined polynomials called nowadays *Bernstein polynomials* as follows.

Definition [**2**]. Let $f : [0, 1] \to \mathbf{R}$. The *Bernstein polynomial* of f is

$$B_n(f; x) := \sum_{k=0}^{n} f\left(\frac{k}{n}\right) \binom{n}{k} x^k (1 - x)^{n-k}, \quad n = 1, 2, \dots .$$

Bernstein proved that, if $f \in C[0, 1]$, then the sequence $\{B_n(f; x)\}$ converges uniformly to $f(x)$ on $[0, 1]$.

Definition. The *Bernstein operator* $B_n : C[0, 1] \to C[0, 1]$ is given by

$$(B_n)f(x) := B_n(f; x), \quad n = 1, 2, \dots .$$

Later it was found that Bernstein polynomials possess many remarkable properties, which made them an area of intensive research. A systematic treatment of the theory of Bernstein polynomials as it was

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