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A CENTRAL LIMIT THEOREM FOR GENERAL WEIGHTED SUMS OF LNQD RANDOM VARIABLES AND ITS APPLICATION

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ABSTRACT. In this paper we derive the central limit theorem for $\sum_{i=1}^{n} a_{ni}\,\xi_i$, where $\{a_{ni},\ 1 \le i \le n\}$ is a triangular array of nonnegative numbers such that $\sup_n \sum_{i=1}^{n} a_{ni}^2 < \infty$, $\max_{1\le i\le n} a_{ni} \to 0$ as $n \to \infty$ and ξ_i's are a linearly negative quadrant dependent sequence. We also apply this result to consider a central limit theorem for a partial sum of a generalized linear process of the form $X_n = \sum_{j=-\infty}^{\infty} a_{k+j}\,\xi_j$.

1. Introduction and results. Lehmann [**8**] introduced a simple and natural definition of positive (negative) dependence: A sequence $\{\xi_i,\ 1 \le i \le n\}$ of random variables is said to be pairwise positive (negative) quadrant dependent (pairwise PQD (NQD)) if, for any real α_i, α_j and $i \ne j$ $P(\xi_i > \alpha_i,\ \xi_j > \alpha_j) \ge (\le) P(\xi_i > \alpha_i)P(\xi_j > \alpha_j)$. Much stronger dependent concepts than PQD and NQD were considered by Esary, Proschan and Walkup [**4**] and Joag-Dev and Proschan [**6**], respectively. A sequence $\{\xi_i,\ 1 \le i \le n\}$ of random variables is said to be associated if, for any real coordinatewise increasing functions f, g on $\mathbf{R}^n$, $\mathrm{Cov}\,(f(\xi_1, \ldots, \xi_n),\ g(\xi_1, \ldots, \xi_n)) \ge 0$ and $\{\xi_i,\ 1 \le i \le n\}$ is said to be negatively associated if, for any disjoint subsets, $A, B \subset \{1, 2, \ldots, n\}$ and any real coordinatewise increasing functions f on $\mathbf{R}^A$ and g on $\mathbf{R}^B$, $\mathrm{Cov}\,(f(\xi_i,\ i \in A),\ g(\xi_i \in B)) \le 0$.

Instead of association (negative association) Newman's [**10**] central limit theorem requires only that positive linear combinations of the random variables are PQD (NQD). The definition of positive (negative) dependence introduced by Newman [**10**] is the following: A sequence $\{\xi_i,\ 1 \le i \le n\}$ of random variables is said to be linearly positive

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