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CONVERGENCE BY NONDISCRETE MATHEMATICAL INDUCTION OF A TWO STEP SECANT'S METHOD

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ABSTRACT. We use the nondiscrete mathematical induction method for the semi-local convergence of a two step secant's iterative scheme on a Banach space. The scheme does not need to evaluate neither any Fréchet derivative nor any bilinear operator, but having a high speed of convergence.

1. Introduction. We consider the problem of approximating locally the unique solution x^* of a nonlinear equation

$$f(x) = 0, \tag{1}$$

where f is a continuous operator defined on the closed convex domain D of a Banach space E_1 with values in a Banach space E_2.

We use the two step Steffensen's method given by

$$\begin{aligned} y_{n+1} &= x_n - [x_n, x_n + \alpha_n(y_n - x_n); f]^{-1} f(x_n), \\ x_{n+1} &= y_{n+1} - [x_n, x_n + \alpha_n(y_n - x_n); f]^{-1} f(y_{n+1}), \quad n \geq 0. \end{aligned} \tag{2}$$

to generate two sequences converging to x^*.

In order to control the stability in practice, the α_n can be computed such that

$$\mathrm{tol}_c << |\alpha_n(y_n - x_n)| \leq \mathrm{tol}_u,$$

where tol_c is related with the computer precision and tol_u is a free parameter.

Here $[x, y; f] \in L(E_1, E_2)$ is a divided difference of order one for the operator f on the points $x, y \in D$. If f is Fréchet differentiable, then

$$[x, y; f] = \int_0^1 f'(x + t(y - x))\, dt, \quad \text{for all} \quad x, y \in D. \tag{3}$$

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