ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 31, Number 3, Fall 2001

ON THE NUMBER OF PARTITIONS WITH A FIXED LEAST PART

NEVILLE ROBBINS

ABSTRACT. Let $P(n)$, $Q(n)$ denote, respectively, the set of all unrestricted partitions of n and the set of all partitions of n into distinct parts. For $1 \leq j \leq n$, we derive formulas that permit the computation of the number of partitions in $P(n), Q(n)$ respectively whose least part is j.

1. Introduction. If $1 \leq j \leq n$, let $F_j(n)$, $f_j(n)$ denote, respectively, the number of partitions of the natural number n whose least part is j, the number of partitions of n into distinct parts whose least part is j. In this note, we derive formulas for the $F_j(n)$. We also derive recurrences that permit the evaluation of the $f_j(n)$ and present asymptotic formulas for the $f_j(n)$. In addition, we determine the parity of $f_1(n)$.

2. Preliminaries.

Definition 1. Let $p(n)$, $q(n)$ denote, respectively, the numbers of unrestricted partitions of n, partitions of n into distinct parts.

Definition 2. If $1 \leq j \leq n$, let $F_j(n)$ denote the number of partitions of n whose least part is j.

Definition 3. If $1 \leq j \leq n$, let $f_j(n)$ denote the number of partitions of n into distinct parts whose least part is j; let $f_j(0) = 0$.

Definition 4. Let $\omega(k) = k(3k-1)/2$.

(1) $q(n) \equiv 1 \pmod 2$ if and only if $n = \omega(\pm k)$ for some $k \geq 1$.

(2) $q(n) \sim 18^{-1/4}(24n+1)^{-3/4}e^{(\pi\sqrt{48n+2}/12)}$.

Remarks. (1) is well known; (2) was proven by Hagis [**1**].

Received by the editors on April 24, 2000.