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SEMI-DISCRETE GALERKIN APPROXIMATIONS FOR THE SINGLE-LAYER EQUATION ON LIPSCHITZ CURVES

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ABSTRACT. We study a semi-discrete Galerkin method for solving the single-layer equation $\mathcal{V}u = f$ with an approximating subspace of piecewise constant functions. Error bounds in Sobolev norms $\|\cdot\|_s$ with $-1 \leq s < 1/2$ are proven and are of the same order as for the original Galerkin method. The distinctive features of the present work are that we handle irregular meshes and do not rely on Fourier methods. The main assumptions are that the quadrature rule used to approximate the inner product is a composite rule and that the underlying quadrature rule that is mapped to each subinterval has a sufficiently small *Peano constant*.

1. Introduction. The single-layer equation

$$\mathcal{V}u = f \tag{1}$$

is an important boundary integral equation. It arises, for example, in the solution of the Laplace equation on interior or exterior domains. If Ω is a two-dimensional domain with Lipschitz boundary Γ, as we shall assume in this paper, then the single-layer operator $\mathcal{V}$ takes the form

$$\mathcal{V}u(t) := -\frac{1}{\pi}\int_\Gamma \log|t-s|u(s)\,ds = f(t), \quad t \in \Gamma, \tag{2}$$

where $|t-s|$ denotes the Euclidean distance between t and s, and ds is the element of arc length. The curve Γ could, for example, be a polygon or 'curved polygon,' without cusps.

In this paper we study a semi-discrete Galerkin method, or 'qualocation method,' with an approximating subspace of piecewise constant functions. Let S_h be the space of piecewise constant functions on a partition

$$\Gamma = \cup\Gamma_k,$$

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