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SINGULAR PERTURBATIONS IN A NONLINEAR VISCOELASTICITY

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ABSTRACT. A nonlinear equation in viscoelasticity of the form

$$\rho u^\rho_{tt}(t,x) = \phi(u^\rho_x(t,x))_x + \int_{-\infty}^t F(t-s)\phi(u^\rho_x(s,x))_x\,ds + \rho g(t,x) + f(x), \qquad t \geq 0, \quad x \in [0,1], \tag{0.1}$$

$$u^\rho(t,0) = u^\rho(t,1) = 0, \qquad t \geq 0, \tag{0.2}$$

$$u^\rho(s,x) = v^\rho(s,x), \qquad s \leq 0, \quad x \in [0,1], \tag{0.3}$$

(where ϕ is nonlinear) is studied when the density ρ of the material goes to zero. It will be shown that when $\rho \downarrow 0$, solutions u^ρ of the dynamical system (0.1)–(0.3) approach the unique solution w (which is independent of t) of the steady state obtained from (0.1)–(0.3) with $\rho = 0$. Moreover, the rate of convergence in ρ is obtained to be $\|u^\rho - w\|_{L^2} \leq K\sqrt{\rho}$ and $\|u^\rho_x - w_x\|_{L^2} \leq K\sqrt{\rho}$ for some constant K independent of ρ.

1. Introduction. Let us begin with the following quasi-static approximation studied in MacCamy [**11**],

$$u_{tt}(t) = -A(0)g(u(t)) - \int_0^t A'(t-s)g(u(s))\,ds + F(t), \tag{1.1}$$

and

$$0 = -A(0)g(w(t)) - \int_0^t A'(t-s)g(w(s))\,ds + F(t). \tag{1.2}$$

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