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ON A WIENER-HOPF INTEGRAL EQUATION

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1. Introduction. In [**1**] the following perceptive observation is made: "Much of the fascination of Wiener-Hopf theory is the difficulty in obtaining explicit answers in concrete cases." In a private communication from one of the authors of [**1**] the following question was posed, "Determine $\{E_n\}$ such that

$$\sin\left(\frac{\pi}{4}+\theta\right)\sum_{n=0}^{\infty}\frac{E_n}{\beta_n-\theta}+\sin\left(\frac{\pi}{4}-\theta\right)\sum_{n=0}^{\infty}\frac{E_n}{\beta_n+\theta} = \sqrt{2}\frac{\sin\theta}{\theta}\sum_{n=0}^{\infty}\frac{E_n}{\beta_n}, \tag{1}$$

where $\beta_n=(n+3/4)\pi$, $n=0,1,2,\ldots$, and $E_n>0$ for all n."

We choose to show that $\sum_{n=0}^{\infty}E_n/\beta_n=1$, so that (1) should read

$$\sin\left(\frac{\pi}{4}+\theta\right)\sum_{n=0}^{\infty}\frac{E_n}{\beta_n-\theta}+\sin\left(\frac{\pi}{4}-\theta\right)\sum_{n=0}^{\infty}\frac{E_n}{\beta_n+\theta}=\sqrt{2}\frac{\sin\theta}{\theta}. \tag{2}$$

Furthermore, we shall relate the solution of (2) to the solution of the integral equation

$$\int_0^{\infty}(\cosh\theta\cos\theta y-\sinh\theta\sin\theta y)P(y)\,dy=\frac{\sinh\theta}{\theta}. \tag{3}$$

Clearly, if we replace θ by $i\theta$, then (3) becomes

$$\int_0^{\infty}\left(\sin\left(\frac{\pi}{4}+\theta\right)e^{\theta y}+\sin\left(\frac{\pi}{4}-\theta\right)e^{-\theta y}\right)P(y)\,dy=\sqrt{2}\frac{\sin\theta}{\theta}, \tag{4}$$

and if we assume that $P(y)$ admits the series expansion

$$P(y)=\sum_{n=0}^{\infty}E_ne^{-\beta_n y},$$

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