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REDUCTION OF AN INFINITE SYSTEM OF INTEGRODIFFERENTIAL EQUATIONS FOR ELECTRIC CURRENTS ON A LATTICE OF CLOSED CURVES TO A FINITE SYSTEM OF INDEPENDENT PSEUDODIFFERENTIAL EQUATIONS ON A CIRCLE

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ABSTRACT. An infinite system of integrodifferential equations for electric currents on an infinite network of nonintersecting infinitely smooth simple closed curves in the plane is considered. These curves are obtained from a fixed curve by a parallel translation by vectors belonging to a one-dimensional lattice. An incident electromagnetic field is assumed to be harmonically time dependent and to be normally polarized so that the electric currents flow only along the curves. The medium is assumed to be linear and isotropic. It is shown that if the incident electromagnetic field is a T-periodic function on the lattice, then the original system can be reduced to a system of T independent pseudodifferential equations on the unit circle with classic elliptic pseudodifferential operators of order 1 in the Sobolev scale. Another significant outcome of this work is that this reduction allows one to apply the many known powerful methods for the numerical analysis of classic elliptic pseudodifferential equations on the unit circle to the original system.

1. Introduction. It is well known (see, for example, [**1, 2, 3**]) that boundary-value problems for Maxwell's equations in a plane isotropic linear medium in which the boundary consists of a finite number N of conducting nonintersecting infinitely smooth simple closed curves can be reduced to a system of N integrodifferential equations for electric currents along the curves. Harmonic time dependency, normal polarization of the electromagnetic field (i.e., that the vector of electric field strength lies in the plane), and the radiation condition at infinity were assumed. The $N \times N$ matrix integrodifferential operator of the system was proven to be an $N \times N$ matrix classic elliptic pseudodifferential operator of order 1 in the Sobolev scale of N-dimensional

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