JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 7, Number 2, Spring 1995

INTEGRATED RESOLVENT OPERATORS

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ABSTRACT. In this paper we introduce the notion of integrated resolvent operators to study the linear Volterra integrodifferential equation

$$\text{(VE)} \qquad u'(t) = Au(t) + \int_0^t B(t-s)u(s)ds + f(t)$$
$$\text{for } t \in [0,T] \text{ and } u(0) = x,$$

where A is a closed linear operator whose domain is not necessarily dense in a Banach space X, and $\{B(t) : t \geq 0\}$ is a family of linear operators in X with $D(A) \subset D(B(t))$ for $t \geq 0$ and of bounded linear operators from Y into X. Here Y is a Banach space $D(A)$ endowed with the graph norm of A. Roughly speaking, the integrated resolvent operator is the "integral"of the solution to the problem (VE) when the forcing term $f \equiv 0$. Our main purpose is to construct the integrated resolvent operator under the suitable conditions on A and $\{B(t) : t \geq 0\}$. The results obtained are applied to two Cauchy problems :

$$u''(t) - Au'(t) - Bu(t) = f(t)$$
$$\text{for } t \in [0,T],\ u(0) = x \text{ and } u'(0) = y;$$
$$u'(t) = A\left(u(t) + \int_0^t F(t-s)u(s)\,ds\right) + Ku(t) + f(t)$$
$$\text{for } t \in [0,T] \text{ and } u(0) = x.$$

As illustrations of our abstract theory, two concrete examples are given.

1. Introduction. Let X be a Banach space with norm $\|\cdot\|$ and denote by $B(X)$ the set of all bounded linear operators from X into itself. This paper is concerned with the linear Volterra integrodifferential

Received by the editors on July 20, 1993, and in revised form on October 15, 1994.
1991 *Mathematics Subject Classification.* Primary 45N05, Secondary 44D05.
Key words and phrases. Integrated resolvent operator, resolvent operator, integral solution, weak solution, integrated semigroup.