JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 7, Number 2, Spring 1995

AN ALGORITHM FOR THE NUMERICAL RESOLUTION OF A CLASS OF SINGULAR INTEGRAL EQUATIONS

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ABSTRACT. We consider a class of integral equations of Volterra type with constant coefficients containing a logarithmic difference kernel. This equation can be transformed into an equivalent singular equation of Cauchy type which allows us to give the explicit formula for the solution. The numerical method proposed in this paper consists of applying the Lagrange interpolation to the inner Cauchy type singular integral in the latter formula after subtracting the singularity. For the error of this method weighted norm estimates as well as estimates on discrete subsets of knots are given. The paper concludes with some numerical examples.

1. Introduction. In this paper we consider the following integral equation

$$a\int_{-1}^{x} g(t)\,dt + \frac{b}{\pi}\int_{-1}^{1} g(t)\log|x-t|\,dt = f(x), \qquad -1 < x < 1, \tag{1.1}$$

under the hypothesis $a, b \in R$, $a^2+b^2=1$, and $f(x) \in C^{p+\lambda}([-1,1])$, $p \geq 1$, and $0 < \lambda \leq 1$, where $C^{p+\lambda}(A)$ is the class of the functions that have p continuous derivatives in A and the p-th derivative is in the space $\mathrm{Lip}_{\lambda}A$, i.e.,

$$\mathrm{Lip}_{\lambda}A := \left\{ f \in C^0(A) : \sup_{x\neq y\in A} \frac{|f(x)-f(y)|}{|x-y|^{\lambda}} < \infty \right\}.$$

In case $a=0$ equation (1.1) coincides with Carleman's equation [**1**]. The integral equation (1.1) has also been considered in [**8**] but uses a different approach.

Here, with the aid of the relation

$$h(x) = \int_{-1}^{x} g(t)\,dt - h_0\frac{1+x}{2} \tag{1.2}$$

Received by the editors on December 1, 1994.