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A MODIFIED DISCRETE SPECTRAL COLLOCATION METHOD FOR FIRST KIND INTEGRAL EQUATIONS WITH LOGARITHMIC KERNEL

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ABSTRACT. Here we propose a modification of the discrete Galerkin method considered by Atkinson for Symm's equation with the logarithmic kernel. Our method has the computational complexity of the trigonometric collocation but it still retains the stability and the convergence properties of the original trigonometric Galerkin method. In particular, the method can be applied with any data having arbitrarily bad non-smoothness. Numerical experiments confirm our results.

1. Introduction. The essence of this paper consists of a simple remark on the discrete Galerkin method discussed by Atkinson in [**1**]. Atkinson considers a fully discrete approximation of the spectral Galerkin method [**3**] for Symm's integral equation with the logarithmic kernel. The method in [**1**] can also be viewed as a further discretization of the trigonometric collocation method. If the solution is smooth, this scheme retains the excellent convergenge properties of the trigonometric Galerkin method, but yields a low order convergence, or in the worst case is not at all applicable, if the solution (or equivalently the data) is not smooth. To be more precise, the minimal smoothness requirement is the continuity of the given right-hand side function.

Here we propose a slight modification of Atkinson's discretization. In our approach the given data is replaced by its L^2-projection onto the subspace of the trigonometric functions. We will show that this modification preserves all the convergence properties of the original trigonometric Galerkin method. In particular, it has optimal order convergence in the full range of the related Sobolev spaces. Moreover, the coefficient matrix remains same as in Atkinson's method. Thus, if the Fourier coefficients of the right-hand side are easy to calculate, the computational cost is very close to that of [**1**]. By using the well-known cosine transformation [**6, 7**] our modification carries also over

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