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THE MODIFIED QUADRATURE METHOD FOR LOGARITHMIC-KERNEL INTEGRAL EQUATIONS ON CLOSED CURVES

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ABSTRACT. Here we discuss the convergence of the modified quadrature method in the approximate solution of boundary integral equations of the first kind with logarithmic kernel. The method consists of regularization of the kernel together with trapezoidal approximation of the integral. We are able to prove convergence of the order $O(h^3)$ for smooth solutions. Numerical experiments confirm our theoretical results.

1. Introduction. Because of its importance in many areas of mathematical physics, the numerical solution of the integral equation

$$-\frac{1}{\pi}\int_\Gamma v(y)\ln|x-y|\,ds_y = g(x), \quad x\in\Gamma \tag{1.1}$$

on a closed curve $\Gamma \subset \mathbf{R}^2$ has attained considerable attention. Remaining with smooth curves we observe that the basic Galerkin- and collocation methods (using splines or trigonometric functions as trial functions) [**3, 4, 5, 10, 13, 16, 17, 21**] and various modifications of these methods for (1.1) have been analyzed extensively [**2, 7, 14, 15, 19, 23, 24**]. In the above articles [**5, 7**], and in [**12**] in connection with [**14, 15**], the effect of numerical integration is also taken into account, which means that fully discretized schemes are available. The same is true also for the spline Galerkin- and collocation methods in the works [**9, 18, 25, 26**] where sufficient conditions for accuracy of the numerical integration are found to preserve the convergence properties of the original method. However, the conventional easy-to-implement quadrature methods for (1.1) have not yet been completely analyzed.

Let us briefly review what is known for quadrature methods applied to (1.1). Assume that we simply use (after choosing a parametric representation for the curve Γ) the composite trapezoidal rule for approximating of the integral in (1.1) and set up the quadrature equations by

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