JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 3, Number 4, Fall 1991

DISTRIBUTIONAL SOLUTIONS OF THE WIENER-HOPF INTEGRAL AND INTEGRO-DIFFERENTIAL EQUATIONS

R. ESTRADA AND R.P. KANWAL

ABSTRACT. We present the theory and technique for obtaining the distributional solutions for the Wiener-Hopf integral and integro-differential equations. This is achieved by identifying a class of kernels for which these equations are well defined and are of the Fredholm type. Consequently, the associated operators and their images are of finite dimensions. Furthermore, we define the operators in such a way that the corresponding equations hold at the end points; otherwise, the equations are usually ill-behaved. We illustrate our analysis with the help of various examples.

1. Introduction. The purpose of this article is to study the distributional solution of the integral equations of the type

$$g(x) + \lambda \int_0^\infty k(x-y)g(y)\,dy = f(x), \quad x \geq 0, \tag{1.1}$$

the so-called Wiener-Hopf integral equation.

The integral equations of the Wiener-Hopf type have attracted the attention of researchers for years. Since the work of Wiener and Hopf [**18**] who introduced the complex variable method that bears their names, many authors have studied the various interesting properties of these equations. Among the many contributions, we would like to call the reader's attention to the work of Krein [**10**] who gave a quite complete theory of the equation of the second kind in the space L^1. The article of Talenti [**16**] surveys the history of these equations.

The solution of Wiener-Hopf equations of the first kind in spaces that contain some generalized functions has been studied by Santos and Teixira [**13, 14**]. The generalization of Krein's L^1 theory to Sobolev spaces, spaces that contain some generalized functions, has also been considered [**12,16**]. Vladimirov [**17**] has gone beyond the distributional framework by studying them in the spaces of ultradistributions.

Our aim is to give the solution of Wiener-Hopf integral and integro-differential equations in the standard spaces of distributions. We