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ANALYSIS OF A CHARACTERISTIC EQUATION

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To John Nohel—colleague, collaborator, and friend—in birthday celebration.

ABSTRACT. The characteristic equation

$$z + s + a + c\left[\frac{1 - e^{-(z+s)}}{z + s}\right] = 0,$$

with $s \geq 0$, arises in the analysis of stability of equilibria of some integrodifferential equations which model the spread of infectious diseases. We obtain some results giving conditions on the parameters a and c for which all roots have negative real part, thus implying stability of an equilibrium.

1. The characteristic equation

$$z + a + c\left(\frac{1 - e^{-z}}{z}\right) = 0 \tag{1}$$

analyzed in [**2, 6, 7**] has arisen in a variety of epidemic models which are formulated as integrodifferential equations. Recently, it has arisen in an S-I-R-S model with a nonlinear incidence rate of the form $\beta I^p S$, a recovery rate γI, and temporary immunity to reinfection for a fixed period ω [**6**]. For this model, there is always a disease-free equilibrium; the number of nontrivial equilibria depends on the values of p and $\sigma = \beta/\gamma$. More specifically, if $p < 1$, there is one nontrivial equilibrium. If $p = 1$, there is no nontrivial equilibrium if $\sigma \leq 1$ and one nontrivial equilibrium if $\sigma > 1$. If $p > 1$, there is a critical value σ^* such that there is no nontrivial equilibrium if $\sigma < \sigma^*$, one nontrivial equilibrium if $\sigma = \sigma^*$, and two nontrivial equilibria if $\sigma > \sigma^*$ [**8, 9**].

If births and deaths are introduced in the above model, with birth rate μ and constant total population size, similar results hold for the

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