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TIME ASYMPTOTIC BEHAVIOR OF THE SOLUTION TO A CAUCHY PROBLEM GOVERNED BY A TRANSPORT OPERATOR

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ABSTRACT. The purpose of this paper is to investigate some spectral properties of a time-dependent linear transport equation with diffusive boundary conditions arising in growing cell populations. After deriving the explicit expression of the strongly continuous semigroup $\{U^K(t) : t \geq 0\}$ generated by the streaming operator, we establish the strict singularity of the operator $BU^K(s)B$, $s > 0$, for a wide class of collision operators B. Making use of the Weis theorem, Theorem 4.1, this enables us to estimate the essential type of the transport semigroup from which the asymptotic behavior of the solution is derived.

1. Introduction. In this paper we deal with the well-posedness and the time asymptotic behavior of solutions to transport equations for a sizable class of scattering operators. More precisely, we are concerned with the following initial boundary value problem

$$(1.1)\quad \begin{cases} (\partial\psi)/(\partial t)(\mu,v,t) = -\,v(\partial\psi)/(\partial\mu)(\mu,v,t) - \sigma(v)\psi(\mu,v,t) \\ + \displaystyle\int_0^c r(\mu,v,v')\psi(\mu,v',t)\,dv' \\ A_K\psi(\mu,v,t) = S_K\psi(\mu,v,t) + B\psi(\mu,v,t), \\ \psi(\mu,v,0) = \psi_0(\mu,v), \end{cases}$$

where $\mu \in [0,a]$, v, $v' \in [0,c]$ with $a > 0$ and $c > 0$, S_K denotes the streaming operator and B stands for the collision one (the integral part of A_K). This model describes the number density $\psi(\mu,v,t)$ of cell population as a function of the degree of maturity $\mu \in [0,a]$, $a > 0$, the maturation velocity $v \in [0,c]$, $c > 0$, and the time t. The degree of maturation is defined so that $\mu = 0$ at the birth and $\mu = c$ at mitosis, i.e. cells born at $\mu = 0$ and divided at $\mu = c$. The kernel $r(\mu,v,v')$ is the transition rate. It specifies the transition of cells from the maturation velocity v' to v while $\sigma(v)$ denotes the total transition

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