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PARALLEL METHODS FOR NONSTIFF VIDES

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Dedicated to Professor Phil Anselone on the occasion of his retirement

ABSTRACT. We consider numerical methods for nonstiff initial-value problems for Volterra integro-differential equations. Such problems may be considered as initial-value problems for ordinary differential equations with expensive righthand side functions because each righthand side evaluation requires the application of a quadrature formula. The often considerable costs suggest the use of methods that require only one righthand side evaluation per step. One option is a conventional linear multistep method. However, if a parallel computer system is available, then one might also look for methods with more righthand sides per step but such that they can all be evaluated in parallel. In this paper we construct such parallel methods and we show that on parallel computers they are by far superior to the conventional linear multistep methods which do not have scope for parallelism. Moreover, the (real) stability interval is considerably larger.

1. Introduction. We consider explicit numerical methods for nonstiff initial-value problems (IVPs) for Volterra integro-differential equations (VIDEs) of the form

$$\frac{d\mathbf{y}(t)}{dt} = \mathbf{f}(\mathbf{y}(t), \mathbf{q}(t)), \quad \mathbf{q}(t) := \int_{t_0}^{t} \mathbf{k}(\mathbf{y}(t), \mathbf{y}(x))\, dx, \tag{1.1}$$
$$\mathbf{y}, \mathbf{f}, \mathbf{k} \in \mathbf{R}^d, \quad t_0 \le t \le t_{\text{end}}.$$

Such IVPs may be considered as IVPs for ordinary differential equations (ODEs) with expensive righthand sides (RHSs) because each RHS evaluation requires the evaluation of the integral term $\mathbf{q}(t)$. In the numerical solution of (1.1), the often considerable costs of the RHSs suggest the application of methods that use only one RHS per step, such as in the conventional linear multistep methods, or if a parallel

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