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APPROXIMATION AND COMMUTATOR PROPERTIES OF PROJECTIONS ONTO SHIFT-INVARIANT SUBSPACES AND APPLICATIONS TO BOUNDARY INTEGRAL EQUATIONS

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Dedicated to Professor Phil Anselone

ABSTRACT. The main purpose of the present paper is to prove approximation and commutator properties for projections mapping periodic Sobolev spaces onto shift-invariant spaces generated by a finite number of compactly supported functions. With these prerequisites at hand, and using certain localization techniques, we then characterize the stability of generalized Galerkin-Petrov schemes for solving periodic pseudodifferential equations in terms of elliptic type estimates of the numerical symbol. Moreover, we establish optimal convergence rates for the approximate solutions with respect to the Sobolev norms.

1. Introduction. It is well known that one of the central problems of the numerical analysis for pseudodifferential equations is to find conditions ensuring the stability of the numerical scheme in consideration. One possible approach to stability analysis for variable symbols is a reduction to the case of constant symbols by means of certain localization techniques which could be viewed as a numerical counterpart to the well-known principle of freezing coefficients in the theory of partial differential equations. The main ingredients for applying such techniques are certain *superapproximation* results for the projections defining the numerical schemes.

Of course, the basic idea of localizing techniques has a long history in theory as well as in the numerical analysis of partial differential

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