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ON THE NON-EXPONENTIAL CONVERGENCE OF ASYMPTOTICALLY STABLE SOLUTIONS OF LINEAR SCALAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT. We study the stability of the scalar linear Volterra equation

$$x'(t) = -ax(t) + \int_0^t k(t-s)x(s)\,ds, \quad x(0) = x_0$$

under the assumption that all solutions satisfy $x(t) \to 0$ as $t \to \infty$. It is shown that if k is a continuously differentiable, positive, integrable function which is subexponential in the sense that $k'(t)/k(t) \to 0$ as $t \to \infty$, then $x(t)$ cannot converge to 0 as $t \to \infty$ faster than $k(t)$.

1. Introduction. In this note we consider the asymptotic stability of the scalar linear Volterra integro-differential equation

$$x'(t) = -ax(t) + \int_0^t k(t-s)x(s)\,ds, \quad t > 0, \tag{1}$$

$$x(0) = x_0. \tag{2}$$

In [**6**] Lakshmikantham and Corduneanu asked if all solutions of (1) satisfy $x(t) \to 0$ as $t \to \infty$, whether that convergence is exponentially fast. The question was natural in view of the fact that asymptotic stability of the zero solution of equations with bounded delay implies exponential asymptotic stability of the zero solution. In [**9**] Murakami showed that exponential asymptotic stability does not automatically follow from the property of (uniform) asymptotic stability of the zero solution. His result assumes that $k \in L^1(0,\infty) \cap C[0,\infty)$ and is of

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