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A PHASE FIELD SYSTEM WITH MEMORY: GLOBAL EXISTENCE

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ABSTRACT. In the present paper we analyze a phase field model with memory:

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\text{(PFM)}\quad \begin{cases} u_t + (l/2)\phi_t = \int_{-\infty}^t a_1(t-s)\Delta u(s)\,ds \\ \qquad\qquad\qquad\qquad\qquad (x,t) \in \Omega \times (0,T) \\ \tau\phi_t = \int_{-\infty}^t a_2(t-s)[\xi^2\Delta\phi + (\phi-\phi^3)/\eta + u](s)\,ds \\ \qquad\qquad\qquad\qquad\qquad (x,t) \in \Omega \times (0,T) \\ \mathbf{n}\cdot\nabla u = \mathbf{n}\cdot\nabla\phi = 0 \quad (x,t) \in \partial\Omega \times (0,T) \\ u(x,0) = u_0(x),\ \phi(x,0) = \phi_0(x) \quad x \in \Omega \end{cases}
$$

for $T > 0$, which has been recently proposed [**29**] as a phenomenological model to describe phase transitions in the presence of slowly relaxing internal variables. The system yields motion by mean curvature with memory under suitable assumptions in a sharp interface limit. In the present paper we give a proof of global existence of a weak solution $(u,\phi) \in \mathbf{C}([0,T]; L^2(\Omega) \times H^1(\Omega))$ for (PFM) assuming that Ω is a smooth bounded domain in R^n, $n = 1, 2$, or 3, the kernels $a_1, a_2 \in L^1(R^+)$ are of positive type, the initial data is in $L^2(\Omega) \times H^1(\Omega)$, and the history is in $L^1(-\infty, 0; H^2(\Omega))$ and $L^1(-\infty, 0; H^3(\Omega)) \cap L^5(-\infty, 0; L^6(\Omega))$, respectively. Our methodology combines results from the theory of Volterra integral equations with Galerkin methods and energy estimates. The results presented here were announced in [**26**].

Keywords and phrases. Phase field equations, memory, integro-differential equations, Galerkin methods, phase transitions.