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VECTOR-VALUED TREE MARTINGALES

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ABSTRACT. In this paper, we first introduce some interesting nontrivial tree martingale examples. Second, we show that, if $1 < a < 2$, then X-valued predictable tree martingale operators $S_t^{(a)}(f)$, $\sigma_t^{(a)}(f)$ can be dominated by X-valued predictable tree martingales f in a certain quasi-norm provided the space X is isomorphic to a $2a$-uniformly convex Banach space.

1. Preliminaries and definitions.

Definition 1.1. Let $\mathbf{T}$ be a countable, upward directed index set with respect to the partial ordering $\preceq$ satisfying the following two conditions:

(1) for every $t \in \mathbf{T}$, the set $\mathbf{T}^t := \{u \in \mathbf{T} : u \preceq t\}$ is finite;

(2) for every $t \in \mathbf{T}$, the set $\mathbf{T}_t := \{u \in \mathbf{T} : t \preceq u\}$ is linearly ordered.

Thus $\mathbf{T}$ is a tree set and every nonempty subset of $\mathbf{T}$ has at least one minimum. The succeeding element of $t \in \mathbf{T}$, namely, the minimum element of the set $\mathbf{T}_t - \{t\}$, is denoted by t^+. A tree $\mathbf{T}$ is also a special partially ordered set with respect to the partial ordering $\preceq$.

Let $(\Omega, \mathscr{S}, \mu)$ be a measure space $(\Omega, \mathscr{S})$, equipped with a finite measure μ, and let $L^1(\Omega, \mathscr{S}, \mu)$ be the space of integrable functions that are measurable relative to $\mathscr{S}$. With the help of positive contractive projections in $L^1(\Omega, \mathscr{S}, \mu)$ spaces, the definition of tree martingales shall be given as follows:

Definition 1.2. Let $(P_t, t \in \mathbf{T})$ be a family of positive contractive projections in $L^1(\Omega, \mathscr{S}, \mu)$ spaces. Then for every $f \in L^1(\Omega, \mathscr{S}, \mu)$, the

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