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THE NUMBER OF MINIMAL COMPONENTS AND HOMOLOGICALLY INDEPENDENT COMPACT LEAVES OF A WEAKLY GENERIC MORSE FORM ON A CLOSED SURFACE

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ABSTRACT. On a closed orientable surface M_g^2 of genus g, we consider the foliation of a weakly generic Morse form ω on M_g^2 and show that for such forms $c(\omega) + m(\omega) = g - (1/2)k(\omega)$, where $c(\omega)$ is the number of homologically independent compact leaves of the foliation, $m(\omega)$ is the number of its minimal components, and $k(\omega)$ is the total number of singularities of ω that are surrounded by a minimal component. We also give lower bounds on $m(\omega)$ in terms of $k(\omega)$ and the form rank $\operatorname{rk}\omega$ or the structure of $\ker[\omega]$, where $[\omega]$ is the integration map.

1. Introduction. Consider a closed connected orientable smooth two-dimensional manifold $M = M_g^2$ of genus g. Let ω be a Morse form on M, i.e., a closed 1-form with Morse singularities $\operatorname{Sing}\omega$, locally the differential of a Morse function. This form defines a foliation $\mathcal{F}_\omega$ on $M \setminus \operatorname{Sing}\omega$. A leaf $\gamma \in \mathcal{F}_\omega$ is called compactifiable if $\gamma \cup \operatorname{Sing}\omega$ is compact.

A Morse form is called *generic* if each of its non-compact compactifiable leaves is compactified by a unique singularity [**2**, Definition 9.1]. The set of such forms is dense in any cohomology class [**2**, Lemma 9.2]. The term *generic* introduced in [**2**] is somewhat misleading because the set of such forms is not open. We find it plausible that such forms are the "majority" of Morse forms and thus their properties are in a sense "typical," though we are not aware of any proof of this.

Our results hold for a wider class of forms, which we call *weakly generic*: the requirement for a leaf to be compactified by only one singularity is only applied to the leaves not surrounded by minimal components.

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