

ON BOREL SUMMABILITY AND ANALYTIC FUNCTIONALS

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ABSTRACT. We show that a formal power series has positive radius of convergence if and only if it is uniformly Borel summable over a circle with center at the origin. Consequently, we obtain that an entire function f is of exponential type if and only if the formal power series $\sum_{n=0}^{\infty} f^{(n)}(0)z^n$ is uniformly Borel summable over a circle centered at the origin. We apply these results to obtain a characterization of those Silva tempered ultradistributions which are analytic functionals. We also use Borel summability to represent analytic functionals as Borel sums of their moment Taylor series over the Borel polygon.

1. Introduction. The aim of this note is to characterize those entire functions f which are of exponential type in terms of the Borel summability of the formal power series

$$(1.1) \quad \sum_{n=0}^{\infty} f^{(n)}(0)z^n.$$

We first prove that a formal power series has positive radius of convergence if and only if it is uniformly (B') summable over some circle with center at the origin. Observe that this is the converse to Borel's classical theorem [1, 2, 4]: If a formal power series has positive radius of convergence, then it is uniformly Borel summable on compacts inside the disk of convergence. Apparently, such a converse result has not been given elsewhere before.

We then obtain the desired characterization for entire functions of exponential type, we show that if (1.1) is uniformly (B') summable

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