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EXISTENCE OF POSITIVE PERIODIC SOLUTIONS IN NEUTRAL NONLINEAR EQUATIONS WITH FUNCTIONAL DELAY

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ABSTRACT. We use Krasnoselskii's fixed point theorem to show that the nonlinear neutral differential equation with functional delay

$$x'(t) = -a(t)x(t) + c(t)x'(t - g(t)) + q(t, x(t - g(t)))$$

which arises in the study of the blood cell, has a positive periodic solution. We apply our results to models in biomathematics.

1. Introduction. Motivated by the papers [**16, 18, 22**], and the references therein, we consider the nonlinear neutral differential equation with functional delay

$$x'(t) = -a(t)x(t) + c(t)x'(t - g(t)) + q(t, x(t - g(t))) \tag{1.1}$$

which arises in a food-limited population models (see [**3–6, 7, 9–11**]), [**17**] and blood cell models, (see [**1, 21**]). For system (1.1), there may be a stable equilibrium point of the population. In the case the equilibrium point becomes unstable, a nontrivial periodic solution may exist. Then oscillation of solutions occurs. The existence of such a stable periodic solution is of quite fundamental importance biologically since it concerns the long time survival of species. The study of such phenomena has become an essential part of qualitative theory of differential equations. For historical background, basic theory of periodicity, and discussions of applications of (1.1) to a variety of dynamical models, we refer the interested reader to [**13–15, 17, 18, 21, 23, 26**].

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