

ON THE EXISTENCE OF SOLUTIONS FOR SCHRÖDINGER-MAXWELL SYSTEMS IN $\mathbb{R}^3$

MINBO YANG, FUKUN ZHAO AND YANHENG DING

ABSTRACT. In this paper we discuss the existence of solutions for the following Schrödinger-Maxwell systems

$$\begin{cases} -\Delta\psi + \lambda\psi + b(x)\phi\psi = a(x)|\psi|^{p-1}\psi & \text{in } \mathbb{R}^3, \\ -\Delta\phi = 4\pi b(x)\psi^2 & \text{in } \mathbb{R}^3. \end{cases}$$

Under suitable assumptions on $a(x)$ and $b(x)$, we establish existence results by variational methods.

1. Introduction and main results. In recent years, the following Schrödinger-Maxwell system has been widely considered

$$(1.1) \quad \begin{cases} -\varepsilon^2\Delta\psi + V(x)\psi + b(x)\phi\psi = a(x)|\psi|^{p-1}\psi & \text{in } \mathbb{R}^3, \\ -\Delta\phi = 4\pi b(x)\psi^2 & \text{in } \mathbb{R}^3. \end{cases}$$

This system arises in quantum mechanics and can be used to describe the standing waves of a classical Schrödinger equation interacting with an unknown electromagnetic field [10, 12, 13].

In recent years, a number of papers have contributed to investigating the existence of solutions of (1.1) and the concentration phenomena of these solutions as $\varepsilon \rightarrow 0$. As we know, if $\varepsilon \rightarrow 0$, the laws of quantum mechanics must reduce to those of classical mechanics, and this describes the transition between quantum mechanics and classical mechanics. D'Aprile and Wei [14], Ruiz [21], Ianni [16], Ianni and Vaira [17, 18] studied the semiclassical limit and constructed a family of radial bound states concentrated around a sphere. For other results concerning the semiclassical limit for a single Schrödinger equation, we refer readers to [1, 11, 15, 26] and the references therein.

2010 AMS *Mathematics subject classification.* Primary 35J20, 35J60, 35Q55.
Keywords and phrases. Variational methods, Schrödinger-Maxwell systems, critical points.

The first author was supported by NSFC (11271331, 11101374), the second author by NSFC (11061040) and the third author by NSFC (10831005).

Received by the editors on October 27, 2009, and in revised form on February 22, 2010.

DOI:10.1216/RMJ-2012-42-5-1655 Copyright ©2012 Rocky Mountain Mathematics Consortium