

THE TOPOLOGICAL CENTER OF WEIGHTED SEMIGROUP ALGEBRAS WITH A STRICT TOPOLOGY

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ABSTRACT. For a family of a locally compact semigroup $\mathfrak{S}$ with a weight function ω , we have recently introduced and studied some locally convex topologies τ on the weighted semigroup algebra $M_a(S, \omega)$ and shown that the strong dual of $(M_a(\mathfrak{S}, \omega), \tau)$ can be identified with a Banach space of certain functions on $\mathfrak{S}$. In this paper, we shall be concerned with the second dual of $(M_a(\mathfrak{S}, \omega), \tau)$; using this duality, we first introduce and study an Arens multiplication on the second dual of $(M_a(\mathfrak{S}, \omega), \tau)$. We then investigate the topological center of $(M_a(\mathfrak{S}, \omega), \tau)$ for an extensive class of locally compact semigroups $\mathfrak{S}$. As a consequence, we conclude some results on Arens regularity and strong Arens irregularity of $(M_a(\mathfrak{S}, \omega), \tau)$.

1. Introduction and preliminaries. Throughout this paper, we denote by $\mathfrak{S}$ a locally compact semigroup; that is, a semigroup with a locally compact Hausdorff topology under which the binary operation of $\mathfrak{S}$ is jointly continuous. We also assume that ω is a *weight function* on $\mathfrak{S}$; that is, a real-valued continuous function ω with the properties that $\omega(x) \geq 1$ and $\omega(xy) \leq \omega(x)\omega(y)$ for all $x, y \in \mathfrak{S}$.

Let $M(\mathfrak{S}, \omega)$ denote the Banach space of all complex-valued regular Borel measures μ on $\mathfrak{S}$ for which

$$\|\mu\|_\omega := \int_{\mathfrak{S}} \omega(x) d|\mu|(x) < \infty,$$

and as usual write $M(\mathfrak{S})$ and $\|\mu\|$ for the case where $\omega(x) = 1$ for all $x \in \mathfrak{S}$, where $|\mu|$ denotes the total variation of μ . Then $M(\mathfrak{S}, \omega)$ is the

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