

ON TRANSFER OPERATORS FOR C^* -DYNAMICAL SYSTEMS

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ABSTRACT. The theme of the paper is the question of existence and basic structure of transfer operators for endomorphisms of a unital C^* -algebra. We establish a complete description of non-degenerate transfer operators, characterize complete transfer operators and clarify their role in crossed product constructions. Also, we give necessary and sufficient conditions for existence of transfer operators for commutative systems, and discuss their form for endomorphisms of $\mathcal{B}(H)$ which is relevant to the Kadison-Singer problem.

Introduction. In [10] Exel introduced a notion of *transfer operator* for C^* -dynamical systems as a natural generalization of the corresponding notion from classical dynamics—the Ruelle-Perron-Frobenius operator. His aim was to use transfer operators in a construction of crossed-products associated with irreversible C^* -dynamical systems, and one of the problematic issues was the dependence of his construction on the choice of a transfer operator which usually is not unique. This problem was to some extent circumvented recently by Bakhtin and Lebedev who introduced in [6] a notion of complete transfer operator which, if it exists, is a necessarily unique non-degenerate transfer operator and is a sufficient tool to deal with the most important types of crossed-products, see [4, 17]. Thus, in this context, but also for future potential applications in noncommutative dynamics, cf. [5, 11], it is essential to understand the structure and the relationship between the non-degenerate and complete transfer operators. So far, however, this topic was not thoroughly investigated and our objective is a response to this deficiency.

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