ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 42, Number 3, 2012

ASYMPTOTIC ANALYSIS OF A FAMILY OF POLYNOMIALS ASSOCIATED WITH THE INVERSE ERROR FUNCTION

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ABSTRACT. We analyze the sequence of polynomials defined by the differential-difference equation $P_{n+1}(x) = P_n'(x)+x(n+1)P_n(x)$ asymptotically as $n \to \infty$. The polynomials $P_n(x)$ arise in the computation of higher derivatives of the inverse error function $\operatorname{inverf}(x)$. We use singularity analysis and discrete versions of the WKB and ray methods and give numerical results showing the accuracy of our formulas.

1. Introduction. The error function $\operatorname{erf}(x)$ is defined by [**1**]

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp\left(-t^2\right) dt \tag{1}$$

and its inverse $\operatorname{inverf}(x)$, which we will denote by $\mathfrak{I}(x)$, satisfies $\mathfrak{I}[\operatorname{erf}(x)] = \operatorname{erf}[\mathfrak{I}(x)] = x$. The function $\mathfrak{I}(x)$ appears in several problems of applied mathematics and mathematical physics [**8**].

In [**4**] we considered the function

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2}\, dt \tag{2}$$

and its inverse $S(x)$, satisfying

$$S\left[N(x)\right] = N\left[S(x)\right] = x.$$

2010 AMS *Mathematics subject classification.* Primary 34E20, Secondary 33E30.
Keywords and phrases. Inverse error function, differential-difference equations, singularity analysis, discrete WKB method.
The work of the first author was supported by a Humboldt Research Fellowship for Experienced Researchers from the Alexander von Humboldt Foundation. The work of the second author was supported by NSF grant DMS 05-03745 and NSA grants H 98230-08-1-0102 and H 98230-11-1-0184.
Received by the editors on November 13, 2008, and in revised form on October 12, 2009.
DOI:10.1216/RMJ-2012-42-3-847