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ON ANNIHILATOR IDEALS IN MATRIX NEAR-RINGS

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ABSTRACT. This paper focuses on how the structure of a faithful R-group of a near-ring R determines the ideal structure of the matrix near-ring, $\mathbf{M}_n(R)$, associated with R. Intersections of annihilating ideals of *monogenic* R-groups or $\mathbf{M}_n(R)$-groups are referred to as *Annihilator ideals*. However, it is known that there exist some *non-monogenic* R-groups, say Δ, for which Δ^n is monogenic as an $\mathbf{M}_n(R)$-group. Taking cognizance of these non-monogenic R-groups helps us draw conclusions on relationships between some Jacobson ν-radicals of R and those of $\mathbf{M}_n(R)$, $\nu = 0, s, 2$. In particular, and contrary to Meldrum-Meyer's conjecture in [**9**], it is herein shown that $(J_0(R))^+ \not\subset J_0(\mathbf{M}_n(R))$.

1. Introduction. Relationships between ideals of a near-ring R and ideals of its associated matrix near-ring $\mathbf{M}_n(R)$ has been the subject of a number of research papers on near-rings. For instance, the Jacobson ν-radicals are shown to be related as $(J_\nu(R))^* \supseteq J_\nu(\mathbf{M}_n(R))$ where $\nu = 0, s, 2$, see [**3, 13**]. A similar relationship was also proved in [**5**] for the socle ideals. That is, $(Soi(R))^* \supseteq Soi(\mathbf{M}_n(R))$, where R satisfies the $DCCL$. In order to draw any conclusion on the relationship between $(J_0(R))^+$ and $J_0(\mathbf{M}_n(R))$ one needs to pay attention to non-monogenic R-groups, say Δ, such that Δ^n is a monogenic $\mathbf{M}_n(R)$-group. These non-monogenic R-groups are identified and referred to as R-groups of *ν_n-form*, according to the type-ν of Δ^n as an $\mathbf{M}_n(R)$-group, $\nu = 0, s, \mathcal{K}$. R-groups of ν_n-form are used to construct an example of a near-ring R such that $(J_0(R))^+ \not\subset J_0(\mathbf{M}_n(R))$. This is despite the fact that $(J_s(R))^+ \subseteq J_s(\mathbf{M}_n(R))$ for a near-ring R such that $\mathbf{M}_n(R)$ satisfies the DCCL, see [**4**]. It is because of the R-groups of ν_n-form that we could construct a near-ring, R, such that $J_0(\mathbf{M}_n(R)) \neq J_s(\mathbf{M}_n(R)) \neq J_2(\mathbf{M}_n(R))$ while $J_0(R) = J_s(R) = J_2(R)$.

Throughout this paper R denotes a right-distributive near-ring with multiplicative identity. If the near-ring R satisfies the descending

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