ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 41, Number 5, 2011

CARLESON MEASURES AND A CLASS OF GENERALIZED INTEGRATION OPERATORS ON THE BERGMAN SPACE

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ABSTRACT. In this paper, we consider a linear operator

$$I_{h,\varphi}^{(n)} f(z) = \int_0^z f^{(n)}(\varphi(\zeta))\, h(\zeta)\, d\zeta$$

induced by holomorphic maps h and φ of the open unit disk $\mathbf{D}$, where $\varphi(\mathbf{D}) \subset \mathbf{D}$ and n is a non-negative integer. A complete characterization of when $I_{h,\varphi}^{(n)}$ is bounded on the Bergman space $\mathcal{A}^2$ is established by using Luecking's result for Carleson measures. We also compute upper and lower bounds for the essential norm of this operator on the Bergman space.

1. Introduction. Let $\mathbf{D}$ be the open unit disk in the complex plane $\mathbf{C}$. Throughout this paper, we denote by $H(\mathbf{D})$ the space of holomorphic functions on $\mathbf{D}$. Let $dA(z) = (1/\pi)\, dx\, dy$, where $z = x + iy$, denote the normalized Lebesgue measure on $\mathbf{D}$. Recall that the Bergman space $\mathcal{A}^2$ is a Hilbert space of holomorphic functions on $\mathbf{D}$ with the norm

$$||f||_{\mathcal{A}^2} = \left(\int_{\mathbf{D}} |f(z)|^2 dA(z) \right)^{1/2} < \infty. \tag{1.1}$$

Also, if $f \in \mathcal{A}^2$ and $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is its Taylor series in $\mathbf{D}$, then $||f||_{\mathcal{A}^2}$ may also be defined as

$$||f||_{\mathcal{A}^2} = \sum_{n=0}^{\infty} \frac{|a_n|^2}{n+1}. \tag{1.2}$$

2010 AMS *Mathematics subject classification.* Primary 47B35, 46E10, Secondary 30H05.
Keywords and phrases. Generalized integration operator, Riemann-Stieltjes operator, composition operator, Bergman space.
Received by the editors on March 13, 2008, and in revised form on January 26, 2009.
DOI:10.1216/RMJ-2011-41-5-1711