ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 41, Number 5, 2011

NCF-DISTINGUISHABLITY BY PRIME GRAPH OF $\mathbf{PGL(2, p)}$ WHERE p IS A PRIME

M. KHATAMI, B. KHOSRAVI AND Z. AKHLAGHI

ABSTRACT. Let G be a finite group. The prime graph $\Gamma(G)$ of G is defined as follows. The vertices of $\Gamma(G)$ are the primes dividing the order of G and two distinct vertices p, p' are joined by an edge if there is an element in G of order pp'. Let p be a prime number. In [**4**], the authors determined the structure of finite groups with the same element orders as $PGL(2,p)$, and it is proved that there are infinitely many nonisomorphic finite groups with the same element orders as $PGL(2,p)$. Therefore there are infinitely many nonisomorphic finite groups with the same prime graph as $PGL(2,p)$.

We know that $PGL(2,p)$ has a unique nonabelian composition factor which is isomorphic to $PSL(2,p)$. Let p be a prime number which is not a Mersenne or Fermat prime and $p \neq 11$, 19. In this paper we determine the structure of finite groups with the same prime graph as $PGL(2,p)$ and as the main result we prove that if G is a finite group such that $\Gamma(G) = \Gamma(PGL(2,p))$ and $p \neq 13$, then G has a unique nonabelian composition factor which is isomorphic to $PSL(2,p)$ and if $p = 13$, then G has a unique nonabelian composition factor which is isomorphic to $PSL(2,13)$ or $PSL(2,27)$.

1. Introduction. If n is an integer, then we denote by $\pi(n)$ the set of all prime divisors of n. Let G be a finite group. Denote by $\pi(G)$ the set of primes p such that G contains an element of order p. Also the set of orders of elements of G is denoted by $\pi_e(G)$. This set is closed under divisibility and is uniquely determined by the set $\mu(G)$ of elements in $\pi_e(G)$ which are maximal under the divisibility relation. We denote by $h(G)$, the number of pairwise non-isomorphic groups H with $\pi_e(G) = \pi_e(H)$. The prime graph $\Gamma(G)$ of a group G is defined as a graph with vertex set $\pi(G)$ in which two distinct primes $p, p' \in \pi(G)$ are adjacent if G contains an element of order pp'. Let $t(G)$ be the number

2010 AMS *Mathematics subject classification.* Primary 20D05, 20D60, 20D08.
Keywords and phrases. Prime graph, simple group, composition factors, linear group.
Received by the editors on July 19, 2008, and in revised form on January 19, 2009.
DOI:10.1216/RMJ-2011-41-5-1523 Copyright ©2011 Rocky Mountain Mathematics Consortium