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NECESSARY AND SUFFICIENT CONDITIONS FOR THE OSCILLATION OF A FIRST-ORDER NEUTRAL DIFFERENTIAL EQUATION OF EULER FORM

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ABSTRACT. In this paper we give a necessary and sufficient condition for the oscillation of the first-order neutral differential equations of Euler form with variable unbounded delays

$$\frac{d}{dt}\left(x(t) - cx(\alpha t)\right) + \frac{1}{t}\sum_{i=1}^{n} p_i x(\beta_i t) = 0, \quad t \geq t_0 > 0,$$

where $0 \leq c < 1$, $0 < \alpha < 1$, $0 < \beta_i < 1$, $\mathrm{p}_i > 0$, $i = 1, 2, \ldots, n$. Some relevant results in the literature are also extended and improved.

1. Introduction. The oscillation theory of delay differential equations and neutral differential equations has drawn much attention in recent years. This is evidenced by extensive references in books of Györi and Ladas [**7**], Erbe et al. [**5**] and Ladde et al. [**10**].

The oscillation of all solutions of neutral differential equation with constant delays and constant coefficients of the form

$$(x(t) - cx(t-\tau))' + \sum_{i=1}^{n} p_i x(t - \tau_i) = 0, \quad t \geq t_0, \tag{1.1}$$

where $0 \leq c < 1$, $\tau, \tau_i, p_i \in (0, \infty)$, $i = 1, 2, \ldots, n$, has been investigated by many authors. See, for example, [**4, 8, 9, 12**] and the references cited therein. In particular, the following well-known oscillation results are established.

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