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BLOWING-UP PROPERTIES OF THE POSITIVE PRINCIPAL EIGENVALUE FOR INDEFINITE ROBIN-TYPE BOUNDARY CONDITIONS

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ABSTRACT. In this paper, we consider the positive principal eigenvalue for some linear elliptic eigenvalue problem with Robin-type boundary conditions having indefinite coefficients, where its asymptotic behavior for indefinite varying weights is investigated. The aim of this paper is to study necessary and sufficient conditions for the positive principal eigenvalue to blow up to infinity. The analysis is based on variational characterization of the positive principal eigenvalue.

1. Introduction and results Let Ω be a bounded domain of $\mathbf{R}^N$, $N \geq 1$, with smooth boundary $\partial\Omega$. This paper is devoted to the study of the following Robin-type eigenvalue problem with indefinite weights.

$$\begin{cases} -\Delta\varphi = \lambda g(x)\varphi & \text{in } \Omega, \\ \dfrac{\partial\varphi}{\partial\mathbf{n}} = \lambda h(x)\varphi & \text{on } \partial\Omega. \end{cases} \tag{1.1}$$

Here, $\Delta = \sum_{j=1}^N \partial^2/\partial x_j^2$ is the usual Laplacian in $\mathbf{R}^N$, λ is a real eigenvalue parameter, $g \in L^\infty(\Omega)$, $h \in W^{1-(1/p),\,p}(\partial\Omega)$ for any $p > 1$, and $\mathbf{n}$ is the unit outer normal to $\partial\Omega$. By $L^p(\Omega)$, $1 \leq p \leq \infty$, we denote the usual Lebesgue space with norm $\|\cdot\|_p$, by $W^{m,p}(\Omega)$, $m = 1, 2, 3, \dots, p > 1$, the usual Sobolev space with norm $\|\cdot\|_{m,p}$, and by $W^{1-(1/p),\,p}(\partial\Omega)$, $p > 1$, the set of traces on $\partial\Omega$ of functions in $W^{1,p}(\Omega)$, equipped with norm $\|\cdot\|_{1-(1/p),\,p,\,\partial\Omega}$. It is well known ([**1**, Theorem 7.53]) that the trace operator T defined by $Tu = u|_{\partial\Omega}$ is an isomorphism and a homeomorphism of $W^{1,p}(\Omega)$ onto $W^{1-(1/p),\,p}(\partial\Omega)$

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