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AN EXPLICIT EXAMPLE CONCERNING THE INVARIANT SUBSPACE PROBLEM FOR BANACH SPACES

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ABSTRACT. We simplify the negative solution to the invariant subspace problem for Banach spaces. Developing the ideas of Read, we give an explicit example of a continuous linear operator on the Banach space l_1 without nontrivial closed invariant subspaces.

1. Introduction. By a *space* we mean a linear manifold over the field $\mathbf{K}$ of real or complex numbers and by an *operator* we understand a linear mapping. Let T be an operator on a space E. A subspace M of E is a nontrivial invariant subspace of T if $\{0\} \neq M \neq E$ and $T(M) \subset M$. One of the most famous problems of operator theory is the invariant subspace problem for Hilbert spaces. It asks whether every continuous operator on an infinite-dimensional (i.d.) separable Hilbert space has a nontrivial closed invariant subspace. This problem is still open. A vast literature exists dedicated to the invariant subspace problem for various important classes of Banach spaces and continuous operators. The invariant subspace problem for complex Banach spaces solved negatively Enflo [**2**] and Read [**5, 6, 7**]. Enflo constructed an i.d. separable Banach space X and a continuous operator on X without nontrivial closed invariant subspaces. The paper containing this very difficult example was submitted for publication in the Acta Mathematica in 1981. It was accepted in 1985 and it appeared in 1987. In the meantime, Read also constructed a counterexample and submitted it for publication in the Bulletin of the London Mathematical Society in 1983. The paper appeared in 1984 [**3**]. A shorter version of this proof was published by Read in 1986 [**5**]. He also constructed continuous operators on Banach spaces l_1 and c_0 without nontrivial

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