ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 40, Number 2, 2010

THE cd-INDEX OF THE POSET OF INTERVALS AND E_t-CONSTRUCTION

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ABSTRACT. Given a graded poset P, let $I(P)$ denote the associated poset of intervals and $E_t(P)$ the poset obtained from P by the E_t-construction of Paffenholz and Ziegler [**7**]. We analyze how the **ab**-index behaves under those operations and prove that its change is expressed in terms of certain, quite explicit, recursively defined linear operators. If the poset P is Eulerian, the recursive relations for those linear operators are interpreted inside the coalgebra spanned by **c** and **d**. We use these relations to prove that the **cd**-index of the dual of the poset of intervals of the simplest Eulerian poset is the same as the **cd**-index of appropriate Tchebyshev poset defined by Hetyei in [**5**].

1. Introduction. Throughout this paper, we will consider graded posets with rank function r. We refer to [**8**] as a good general reference for the poset terminology. For a poset P of rank $n+1$ and $S \subseteq [n] = \{1, 2, \dots, n\}$, let $f_S(P)$ denote the number of chains $x_1 < x_2 < \cdots < x_{|S|}$ such that $S = \{r(x_1), r(x_2), \dots, r(x_{|S|})\}$. The sequence $(f_S(P))_{S \subseteq [n]}$ is called the *flag f-vector* of P.

The flag f-vector of P can be encoded as a homogenous noncommutative polynomial in the variables $\mathbf{a}$ and $\mathbf{b}$. Let P be a poset of rank $n+1$. To every chain

$$c = \left\{\hat{0} < x_1 < x_2 < \cdots < x_k < \hat{1}\right\}$$

of P we associate a *weight* $\mathrm{wt}\,(c) = w_1 w_2 \cdots w_n$ where

$$w_i = \begin{cases} \mathbf{b} & \text{if } i \in \{r(x_1), r(x_2), \dots, r(x_k)\}; \\ \mathbf{a} - \mathbf{b} & \text{otherwise.} \end{cases}$$

2010 AMS *Mathematics subject classification.* Primary 06A07, 05E99.
Keywords and phrases. **ab**-index, **cd**-index, Eulerian posets, poset of intervals, E_t-construction, Tchebyshev posets.
Received by the editors on May 29, 2006, and in revised form on October 15, 2007.
DOI:10.1216/RMJ-2010-40-2-527