

Λ -ABSOLUTE CONTINUITY

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ABSTRACT. A characterization of functions continuous in Λ -variation is given based on the decomposition of a local Λ -variation. A natural generalization of absolutely continuous functions is introduced, and several characterizations of it are stated. Important relationships between various classes and subclasses of functions of bounded Λ -variation are studied.

The concept of bounded harmonic variation arose naturally from the theory of Fourier series and has many applications to it. The most important among them is the following theorem that furnishes the same conclusions as the Dirichlet-Jordan theorem.

Waterman's Test. *If f is of bounded harmonic variation, then*

- (i) $S[f](x)$ converges to $(f(x+) + f(x-))/2$ pointwise;
- (ii) $S[f]$ converges to f uniformly on every closed interval of points of continuity.

The test was proven in [18] where also a general concept of Λ -variation of a function was introduced. Later Waterman found a direct proof of his test, not resting on the Lebesgue test, see [21, 23]. The Waterman test includes all other tests that yield Dirichlet-Jordan type conclusions: the classical Dirichlet-Jordan test, the Salem tests and the Garsia-Sawyer test. Waterman's test is much more convenient to use than the Lebesgue test since the second condition of the latter test corresponds to no simple property of a tested function.

There are numerous other applications of the concept of Λ -variation to the theory of Fourier series. The earlier results have been collected in [1].

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